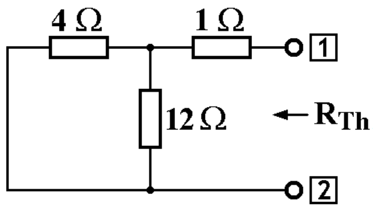
SOLUTION SET II

EXERCISE II.1 : THÉVENIN'S EQUIVALENT CIRCUIT

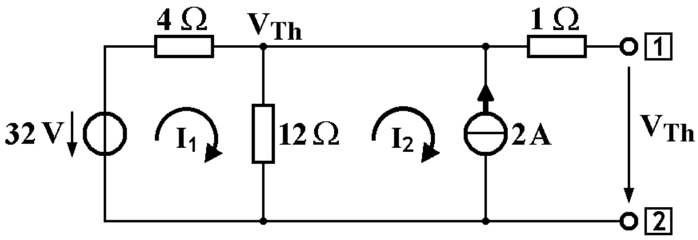


A. We find the resistance of Thévenin R_{Th} by replacing the voltage source with a short circuit and the current source with an open circuit, as shown in the figure on the right.

Thévenin's resistance is obtained by grouping the resistors of 4 Ω and 12 Ω which are in parallel, then in series with the one of 1 Ω :

$$R_{Th} = \frac{4 \times 12}{4+12} + 1 = 4\,\Omega$$

B. Determination of the voltage of Thévenin V_{Th} as shown in the figure below.



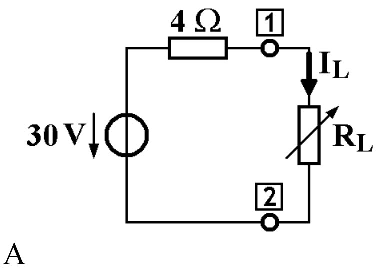
In the current mesh I_1, the voltage equation is expressed as :

$$-32 + 4\,I_1 + 12\,(I_1 - I_2) = 0$$

As $I_2 = -2\text{ A} \quad \Rightarrow \quad I_1 = 0{,}5\text{ A}$

Thévenin's voltage can be calculated across the resistor of 12 Ω :

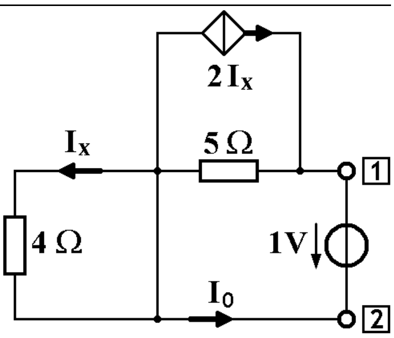
$$V_{Th} = 12 \cdot (I_1 - I_2) = 10 \cdot (0{,}5 + 2) = 30\text{ V}$$



The figure opposite shows Thévenin's equivalent circuit, connected to the load resistors between the terminals $\boxed{1} - \boxed{2}$. Charge current I_L is given by :

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{30}{4 + R_L} \quad \Rightarrow \quad \begin{cases} R_L = 6\,\Omega & \Rightarrow & I_L = 3\text{ A} \\ R_L = 16\,\Omega & \Rightarrow & I_L = 1{,}5\text{ A} \\ R_L = 36\,\Omega & \Rightarrow & I_L = 0{,}75\text{ A} \end{cases}$$

EXERCISE II.2 : NORTON'S THEOREM



A. The Norton resistance is found by replacing the independent voltage source with a short circuit and connecting a 1 V voltage source between the terminals. $\boxed{1} - \boxed{2}$ of the circuit, which gives the diagram of the figure opposite.

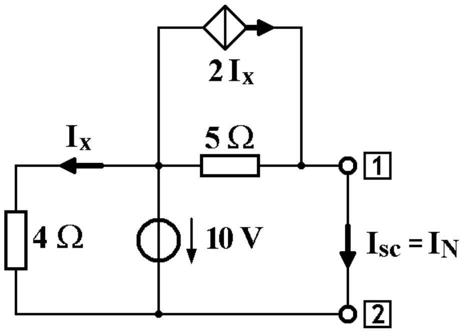
In this diagram :

- The resistor of 4 Ω is short circuited ; we can ignore it and $I_X = 0$.
- The dependent current source, the voltage source and the resistor of 5 Ω are in parallel.

$\Rightarrow \quad I_0 = \frac{1\,V}{5\,\Omega} = 0{,}2\,A \quad \Rightarrow \quad R_N = \frac{U_0}{I_0} \quad \Rightarrow \quad \boxed{R_N = 5\,\Omega}$

B. Norton's current I_N is found by shorting the terminals $\boxed{1} - \boxed{2}$ of the circuit, and calculating the current I_{SC} which is established in this short circuit. In this diagram :

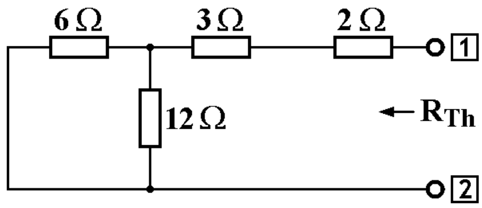
$I_x = \frac{10\,V}{4\,\Omega} = 2{,}5\,A \quad \Rightarrow \quad I_{SC} = \frac{10\,V}{5\,\Omega} + 2 \cdot I_X \quad \Rightarrow \quad \boxed{I_N = 7\,A}$



EXERCISE II.3 : POWER TRANSFER

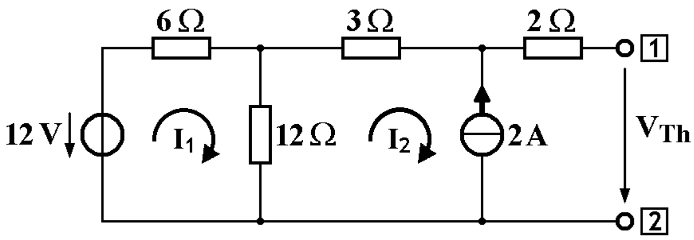
We must find the resistance of Thévenin R_{Th} and the voltage of Thévenin V_{Th}.

A. We find the resistance of Thévenin R_{Th} by replacing the voltage source with a short circuit and the current source with an open circuit, as shown in the figure on the right. The resistors of 6 Ω and 12 Ω are in parallel with each other and in series with the resistors of 3 Ω and 2 Ω :



$R_{Th} = \frac{6 \times 12}{6+12} + 2 + 3 \quad \Rightarrow \quad \boxed{R_{Th} = 9\,\Omega}$

We get the voltage of Thévenin V_{Th} by applying Kirchhoff's law of currents in the meshes of the figure below.



In the current mesh I_1 :

$-12 + 18\,I_1 - 12\,I_2 = 0 \quad$ avec $I_2 = -2\,A \quad \Rightarrow \quad I_1 = -2/3\,A$

In the large mesh formed by the whole circuit :

$-12 + 6\,I_1 + 3\,I_2 + V_{Th} = 0 \quad$ avec $I_2 = -2\,A \quad \Rightarrow \quad V_{Th} = 22\,V$

B. Maximum power transfer P_{max} is reached with $R_L = R_{Th}$ and the voltage V_{Th} between the terminals $\boxed{1}$ and $\boxed{2}$:

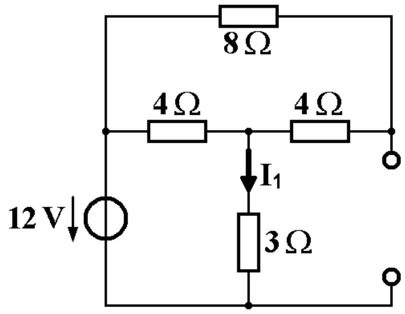
$P_{max} = \frac{V_{Th}^2}{4R_{Th}} \quad \Rightarrow \quad \boxed{P_{max} = 13{,}44\,W}$

EXERCISE II.4 : SUPERPOSITION THEOREM

We have three sources that each contribute to feed current I. Let's call :

- I_1 the component of the current due to the voltage source of 12 V
- I_2 the component of the current due to the voltage source of 24 V
- I_3 the component of the current due to the voltage source of 3 A.

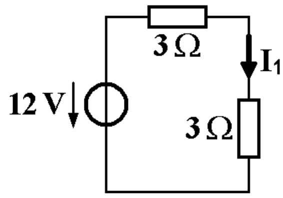
1. Compute I_1 keeping only the 12 V source, as shown in the figure opposite.



In this diagram the resistor of 8 Ω is simply in series with the one of 4 Ω (to the right). These two resistors can therefore be replaced by a total resistance of 12 Ω, which is then in parallel with the resistor of 4 Ω (to the left). The equivalent resistance is given by :

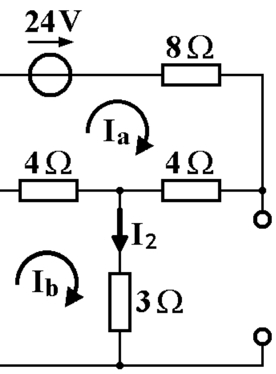
$$R_{éq} = \frac{12 \times 4}{12+4} = 3\,\Omega \tag{1}$$

The diagram above can be simplified as shown in the figure opposite.



$$I_1 = \frac{12\text{ V}}{6\,\Omega} = 2\text{ A} \tag{2}$$

2. Compute I_2 retaining only the 24 V source, as shown in the figure. The analysis of the upper mesh gives :



$$16\,I_a - 4\,I_b + 24 = 0 \tag{3}$$

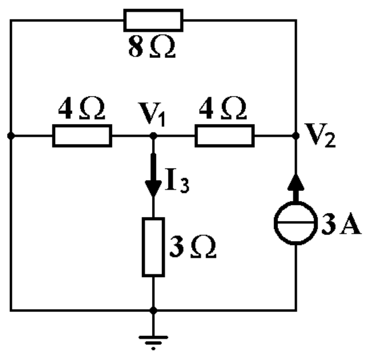
And in the mesh of the current I_b :

$$7\,I_b - 4\,I_a = 0 \tag{4}$$

By substituting (4) in (3) :

$$I_2 = I_b = -1 \tag{5}$$

3. Compute keeping only the source of 3 A, as shown in the figure opposite. The sum of currents at the node V_2 gives :



$$3 = \frac{V_2}{8} + \frac{V_2 - V_1}{4} \quad \Rightarrow \quad 24 = 3\,V_2 - 2\,V_1 \tag{6}$$

And at node V_1 :

$$\frac{V_2 - V_1}{4} = \frac{V_1}{4} + \frac{V_1}{3} \quad \Rightarrow \quad V_2 = \frac{10}{3} V_1 \tag{7}$$

By substituting (7) in (6) we obtain $V_1 = 3$ V :

$$\Rightarrow \quad I_3 = \frac{V_1}{3\,\Omega} = 1\text{ A} \tag{8}$$

Finally, $I = I_1 + I_2 + I_3 \quad \Rightarrow \quad \boxed{I = 2\text{ A}}$